Ocean: #2 Dynamics

Geostrophic and hydrostatic balance

- On the large-scale, water obeys the same fluid dynamics as air
  - Geostrophic balance
  - Hydrostatic balance
- In the ocean, the density change is rather small, and we can take this advantage in writing the momentum equation and hydrostatic balance equation.

Geostrophic and hydrostatic balance

Momentum equations

$$\frac{Du}{Dt} + \frac{1}{\boxed{\rho_{ref}}}\frac{\partial p}{\partial x} - fv = \mathscr{F}_x$$

$$\frac{Dv}{Dt} + \frac{1}{\boxed{\rho_{ref}}}\frac{\partial p}{\partial y} + fu = \mathscr{F}_y$$

constant reference density

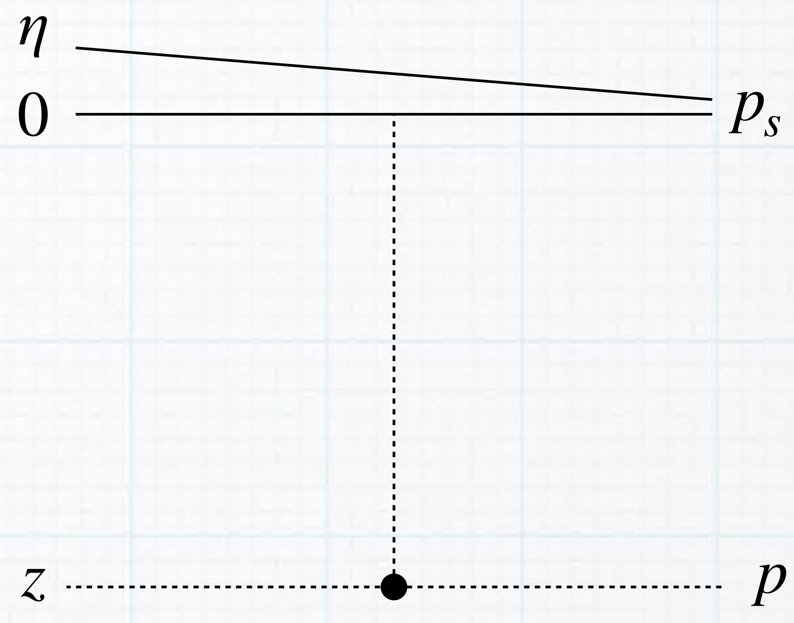
Hydrostatic balance

$$\frac{\partial p}{\partial z} = -g\left(\boxed{\rho_{ref}} + \sigma\right)$$

Geostrophic and hydrostatic balance

Hydrostatic balance in the ocean

$$\frac{\partial p}{\partial z} = -g\left(\rho_{ref} + \sigma\right)$$



$$p(z) = p_s - g\left(\rho_{ref} + \sigma\right)(z - \eta)$$

$$\approx p_s - g\rho_{ref}(z - \eta)$$

- p increases linearly, which is contrasted with the exponential decrease of pressure in the atmosphere.
- Sea level variation can create horizontal pressure gradient at depth.

Geostrophic and hydrostatic balance

- Geostrophic balance in the ocean
  - Typical ocean flow in the subtropical gyre: $U \sim 0.1$ m/s
  - The size of the gyre in north-south direction: $L \sim 2 \times 10^6$ m
  - Coriolis parameter in the midlatitude: $f \sim 10^{-4}$ s^{-1}

$$R_0 = \frac{U}{fL} \sim 10^{-3}$$

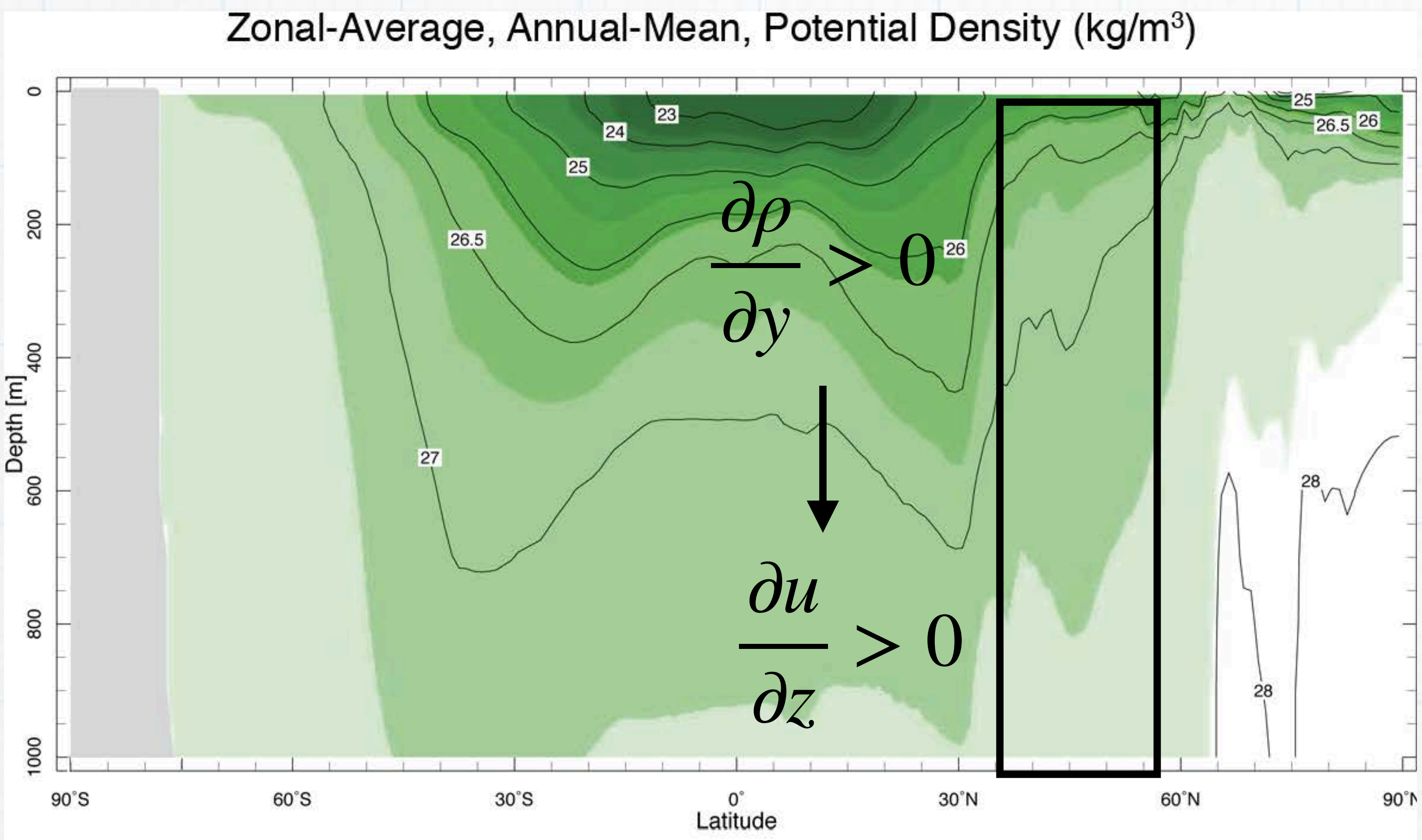
← Much smaller than R_0 in the atmosphere ($O(0.1)$)

→ The geostrophic approximation is valid for the interior of the ocean.

Geostrophic and hydrostatic balance

- Thermal wind balance in the ocean

$$\frac{\partial u}{\partial z} = \frac{g}{f\rho_{ref}}\frac{\partial \sigma}{\partial y} \qquad \frac{\partial v}{\partial z} = -\frac{g}{f\rho_{ref}}\frac{\partial \sigma}{\partial x}$$

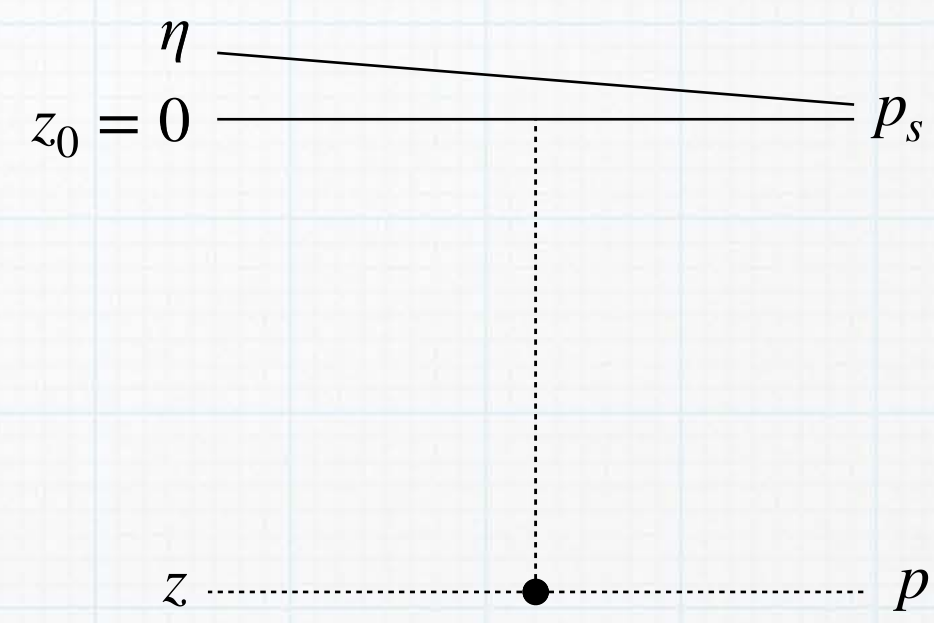


If $u(z) \sim 0$, then what would $u_{surface}$ be?

- $u(1000\ \mathrm{m}) \sim 0$ m/s
- $g = 10$ m/s^2
- $\rho_{ref} = 1000$ kg/m^3
- $\Delta\rho = 1.5$ kg/m^3
- $L = 2000$ km

Ocean surface structure

- On the large-scale, we can use geostrophic balance to understand the relationship between the sea level and the current.



$$p(z) = p_s + \int_z^{\eta} g\rho dz = p_s + g \langle \rho \rangle (\eta - z)$$

$$\frac{1}{\eta - z}\int_z^{\eta} \rho dz$$

treat it as a constant near the surface

$$p(z_0) = p_s + g\rho_{ref}\eta$$

$$u_{g,\, surface} = -\frac{g}{f}\frac{\partial \eta}{\partial y}, \quad v_{g,\, surface} = \frac{g}{f}\frac{\partial \eta}{\partial x}$$

→ geostrophic current just beneath the surface

Ocean surface structure

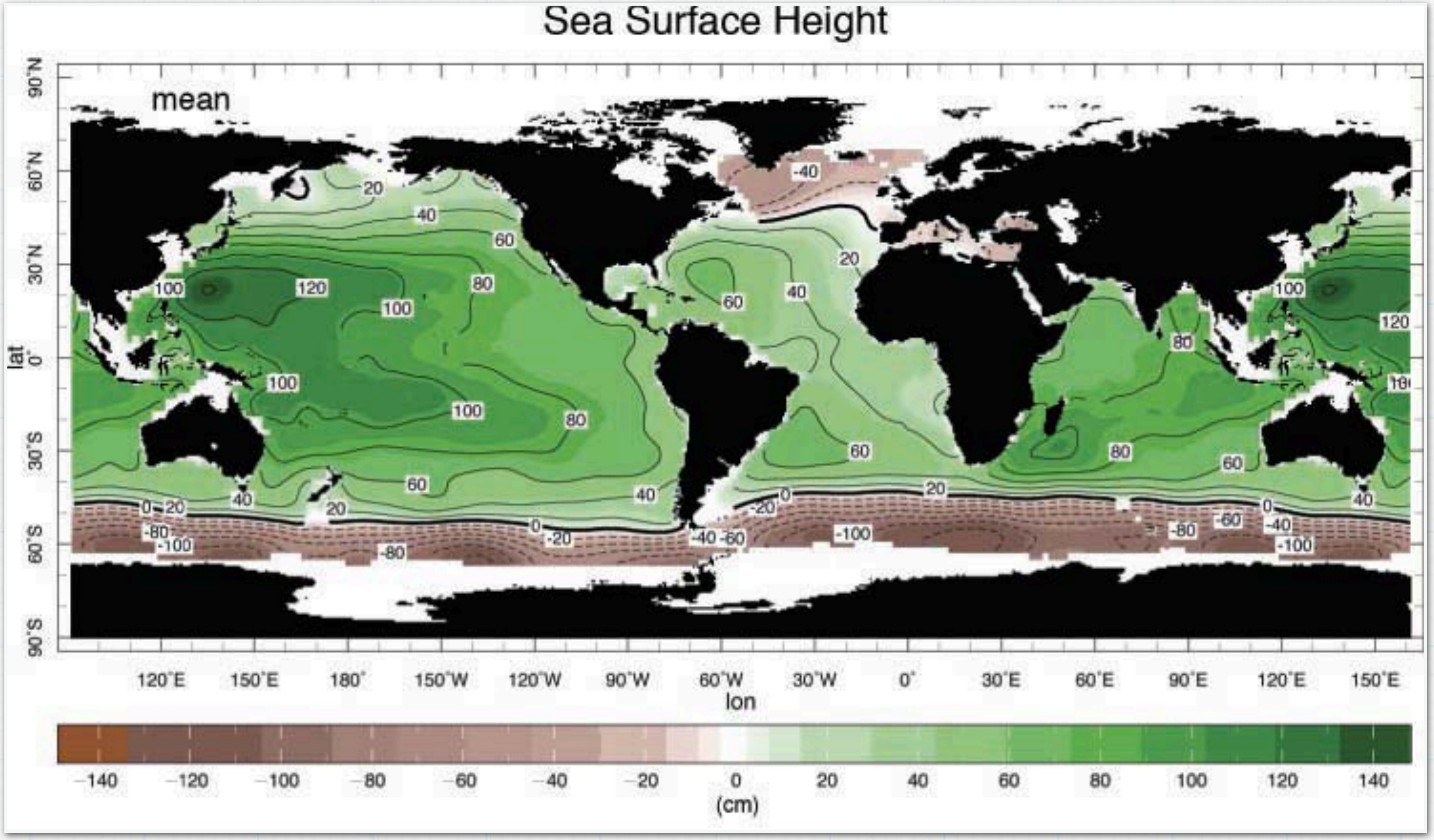
- Estimating sea level changes using the current based on the geostrophic balance

$$\Delta\eta = \frac{fLU}{g} \longrightarrow \Delta\eta \sim O(1\ m)$$

f: $10^{-4}\ \mathrm{s}^{-1}$, L: $1000\ \mathrm{km}$, U: $10\ \mathrm{cm\ s}^{-1}$

g: $10\ \mathrm{m\ s}^{-2}$

Ocean surface structure



Geostrophic current at depth

- At depth, we cannot neglect σ (the variation in density)
- It means that $\nabla \langle \rho \rangle \neq 0$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x}\left[p_s + g\langle\rho\rangle(\eta - z)\right] = g\left[\frac{\partial\langle\rho\rangle}{\partial x}(\eta - z) + \langle\rho\rangle\frac{\partial\eta}{\partial x}\right]$$

$$\frac{\partial p}{\partial y} = g\left[\frac{\partial\langle\rho\rangle}{\partial y}(\eta - z) + \langle\rho\rangle\frac{\partial\eta}{\partial y}\right]$$

$$\longrightarrow$$

$$u_g = -\frac{g}{f\rho_{ref}}\left[\frac{\partial\langle\rho\rangle}{\partial y}(\eta - z) + \langle\rho\rangle\frac{\partial\eta}{\partial y}\right]$$

$$v_g = \frac{g}{f\rho_{ref}}\left[\frac{\partial\langle\rho\rangle}{\partial x}(\eta - z) + \langle\rho\rangle\frac{\partial\eta}{\partial x}\right]$$

Geostrophic current at depth

$$u_g = -\frac{g}{f\rho_{ref}}\left[\frac{\partial \langle\rho\rangle}{\partial y}(\eta - z) + \langle\rho\rangle\frac{\partial \eta}{\partial y}\right]$$

$$v_g = \frac{g}{f\rho_{ref}}\left[\frac{\partial \langle\rho\rangle}{\partial x}(\eta - z) + \langle\rho\rangle\frac{\partial \eta}{\partial x}\right]$$

If there is no horizontal variation in $\langle\rho\rangle$, the geostrophic current is independent of depth.

⬇

Same current at all depth!

⬇

The ocean moves around as a column.

⬅

No vertical motion (2D flow)

⬇

BUT we observe that the ocean current at depth is slower than the surface flow.

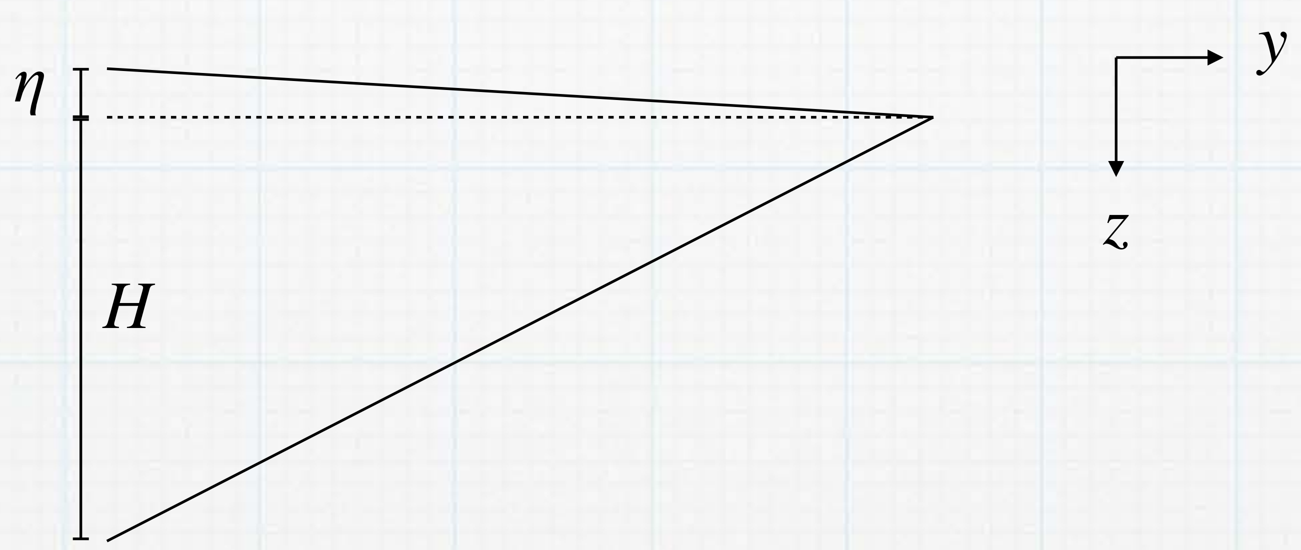
⬇

The first term should reduce the effect from the sea level difference.

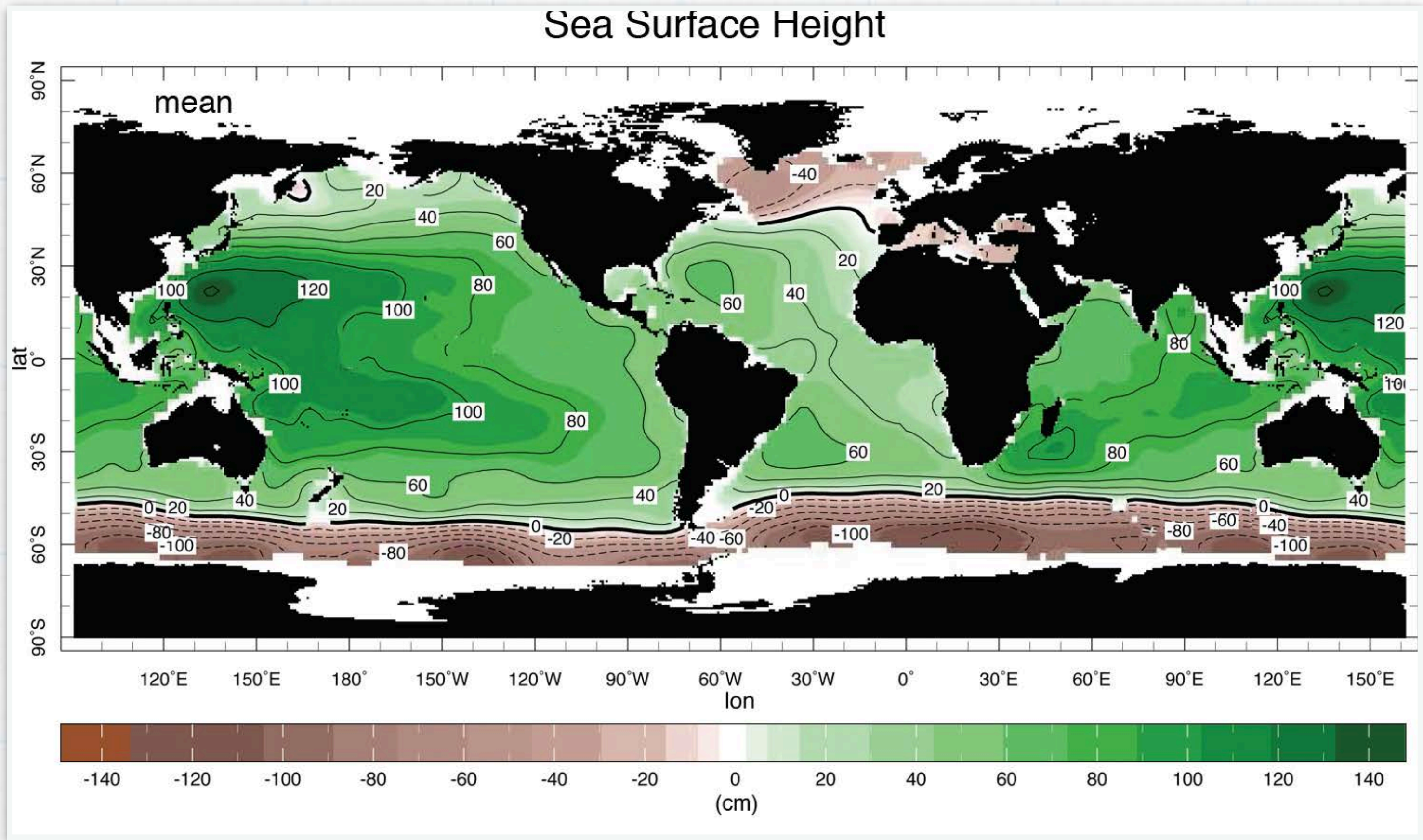
Geostrophic current at depth

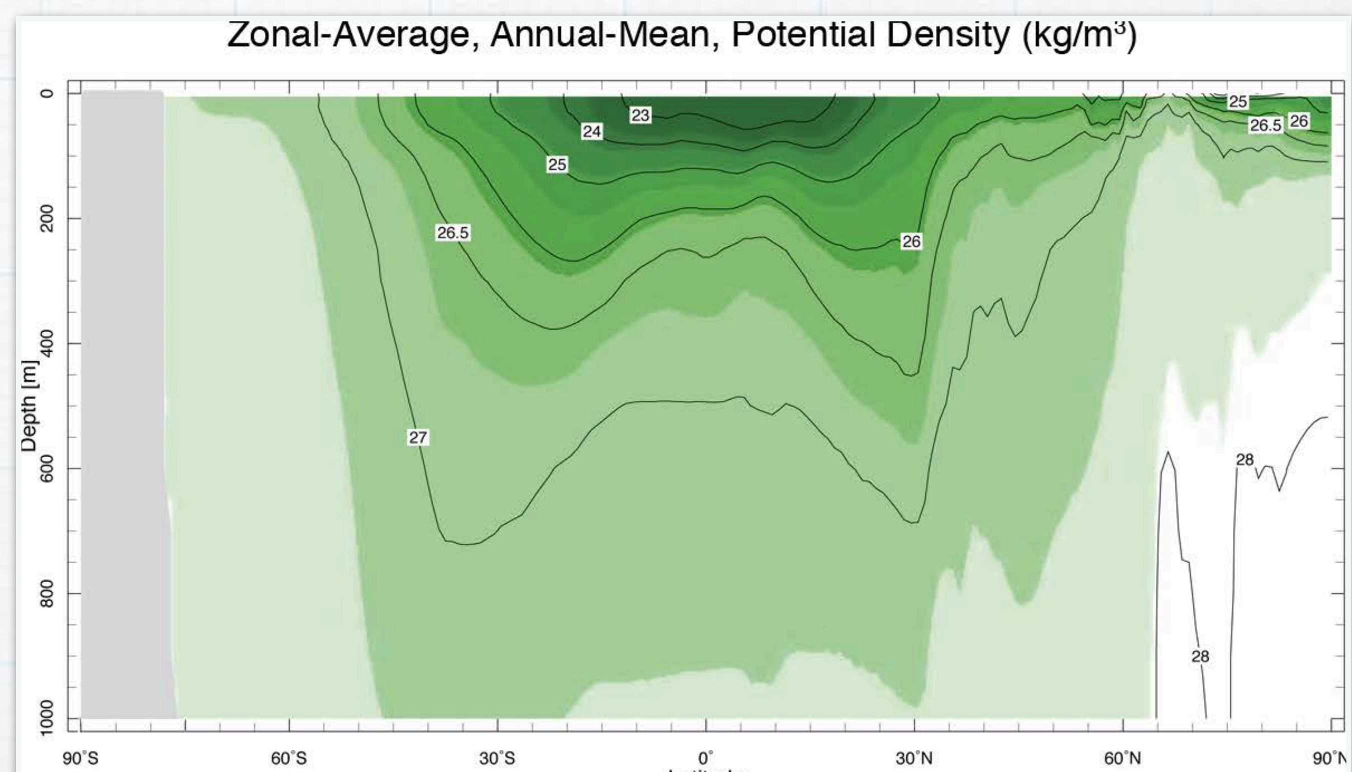
- How much would interior density surface need to tilt to result in the near-zero geostrophic current at $z = H$?
  - Then the pressure gradient should be close to zero

$$\frac{\partial \langle \rho \rangle}{\partial y} H = - \langle \rho \rangle \frac{\partial \eta}{\partial y}$$



Geostrophic current at depth





Steric effects

- From

$$\langle () \rangle = \frac{1}{H+\eta}\int_{-H}^{\eta} ()\, dz$$

$$\frac{\partial \langle \rho \rangle}{\partial y} H = -\langle \rho \rangle \frac{\partial \eta}{\partial y}$$

$$\frac{\Delta \eta}{H} \approx \frac{\nabla \rho}{\rho} \approx \left(\alpha_T \left\langle T - T_0 \right\rangle - \beta_S \left\langle S - S_0 \right\rangle\right)$$

$\longrightarrow$ Warmer and fresh water tends to expand

- Over the top kilometer, $\left\langle T - T_0 \right\rangle \approx 10$ degC and $\left\langle S - S_0 \right\rangle \approx 0.5$ psu.

- If $\alpha_T = 2 \times 10^{-4}$ and $\beta_S = 7.6 \times 10^{-4}$, then
  - $$\frac{\Delta \eta}{H} \approx \left(2 + (-0.38)\right) \times 10^{-3}$$

The dynamic method

- The thermal wind relation is the key to estimate the ocean current using T and S.

$$u_g(z) - u_g(z_1) = \frac{g}{f}\int_{z_1}^{z} \frac{1}{\rho_{ref}}\frac{\partial \sigma}{\partial y}dz = \frac{g}{f}\frac{\partial D}{\partial y},$$

$$\text{where } D = \int_{z_1}^{z} \frac{\sigma}{\rho_{ref}}dz$$

- D is known as the dynamic height.
- T, S measurement $\rightarrow \sigma \rightarrow D \rightarrow u_g$: dynamic method
- $u_g(z_1)$ has to be given to complete the calculation.
    - Level of no motion
    - Current measurement at the surface

The dynamic method